

- Questions

- ▼ Mergesort

- divide & conquer

- ▼ merge function

```
def merge(left, right):  
  
    left_cursor, right_cursor = 0, 0  
    merged = []  
    while left_cursor < len(left) and right_cursor < len(right):  
  
        # Sort each one and place into the result  
        if left[left_cursor] <= right[right_cursor]:  
            merged.append(left[left_cursor])  
            left_cursor += 1  
        else:  
            merged.append(right[right_cursor])  
            right_cursor += 1  
  
    for left_cursor in range(left_cursor, len(left)):  
        merged.append(left[left_cursor])  
  
    for right_cursor in range(right_cursor, len(right)):  
        merged.append(right[right_cursor])  
  
    return merged
```

- ▼ recursively sort left and right halves, then merge

```
def merge_sort(arr):  
    mid = len(arr) // 2  
    # Perform merge_sort recursively on both halves  
    left, right = merge_sort(arr[:mid]), merge_sort(arr[mid:])  
  
    # Merge each side together  
    return merge(left, right)
```

▼ base case

```
# The last array split
if len(arr) <= 1:
    return arr
```

▼ analysis

- merge operation is $O(n)$

▼ how many merges?

- Number of times n can be divided by 2 before base case— $\log_2(n)$
- Gives us $O(n\log_2(n))$, which we will compare to

▼ diagram merge_sort([70, 68, 93, 9, 63, 30]), left = [68, 70, 93], right = [9, 30, 63]

▼ merge_sort([70, 68, 93]), left = [70], right = [68, 93]

- merge_sort([70]), base case

▼ merge_sort([68, 93]), left = [68], right = [93]

- merge_sort([68]), base case
- merge_sort([93]), base case

▼ merge_sort([9, 63, 30]), left = [9], right = [30, 63]

- merge_sort([9]), base case

▼ merge_sort([63, 30]), left = [63], right = [30]

- merge_sort([63]), base case
- merge_sort([30]), base case

• compare timing

▼ Scenarios

▼ in-place vs not

- requires $O(1)$ extra space, usually modifies original array
- insertion sort is in-place, merge sort (as we implemented it) is not

▼ stability

- equal elements remain in the same relative order before and after sorting

▼ essential if we want to sort on one attribute and then another

- list of people, sort by age then by marital status
- both merge sort and insertion sort are stable, selection sort is not

▼ streaming data

- insertion sort is great for sorting data as it comes in ($O(n)$ to insert a single element), merge sort we have to run the whole sort again

▼ The ideal sorting algorithm would have the following properties:

- Stable: Equal keys aren't reordered.
- Operates in place, requiring $O(1)$ extra space.
- Worst-case $O(n \cdot \lg(n))$ key comparisons.
- Worst-case $O(n)$ swaps.
- Adaptive: Speeds up to $O(n)$ when data is nearly sorted or when there are few unique keys.
- There is no algorithm that has all of these properties, and so the choice of sorting algorithm depends on the application.

▼ Visualizations

- <https://www.toptal.com/developers/sorting-algorithms>
- <https://www.youtube.com/user/AlgoRhythms>

- hand back quizzes (median 31),
reflections due last day of class, 2nd
peer evaluation due last day of class