

# Lecture #11: Joins Algorithms

15-445/645 Database Systems (Fall 2020)

<https://15445.courses.cs.cmu.edu/fall2020/>

Carnegie Mellon University

Prof. Andy Pavlo

## 1 Joins

---

The goal of a good database design is to minimize the amount of information repetition. This is why tables are composed based on normalization theory. Joins are therefore needed to reconstruct the original tables.

This class will cover **inner equijoin** algorithms for combining two-tables. An *equijoin* algorithm joins tables where keys are equal. These algorithms can be tweaked to support other joins.

### Operator Output

For a tuple  $r \in R$  and a tuple  $s \in S$  that match on join attributes, the join operator concatenates  $r$  and  $s$  together into a new output tuple.

In reality, contents of output tuples generated by a join operator varies. It depends on the DBMS's query processing model, storage model, and the query itself. There are multiple approaches to the contents of the join operator output.

- **Data:** This approach copies the values for the attributes in the outer and inner tables into tuples put into an intermediate result table just for that operator. The advantage of this approach is that future operators in the query plan never need to go back to the base tables to get more data. The disadvantage is that this requires more memory to materialize the entire tuple.  
The DBMS can also do additional computation and omit attributes which will not be needed later in the query to further optimize this approach.
- **Record Ids:** In this approach, the DBMS only copies the join keys along with the record ids of the matching tuples. This approach is ideal for column stores because the DBMS does not copy data that is not needed for the query. This is called *late materialization*.

### Cost Analysis

The cost metric used here to analyze the different join algorithms will be the number of disk I/Os used to compute the join. This includes I/Os incurred by reading data from disk as well as writing any intermediate data out to disk. Note that only I/Os from computing the join are considered, while I/O incurred when outputting the result is not. This is because the output for any algorithm will be the same, so the output cost will not change among different algorithms.

Variables used in this lecture:

- $M$  pages in table  $R$  (Outer Table),  $m$  tuples total
- $N$  pages in table  $S$  (Inner Table),  $n$  tuples total

In general, there will be many algorithms/optimizations which can reduce join costs in some cases, but no single algorithm which works well in every scenario.

## 2 Nested Loop Join

---

At a high-level, this type of join algorithm is comprised of two nested for loops that iterate over the tuples in both tables and compares each unique of them. If the tuples match the join predicate, then output them. The table in the outer for loop is called the *outer table*, while the table in the inner for loop is called the *inner table*.

The DBMS will always want to use the “smaller” table as the outer table. Smaller can be in terms of the number of tuples or number of pages. The DBMS will also want to buffer as much of the outer table in memory as possible. It can also try to leverage an index to find matches in inner table.

### Simple Nested Loop Join

For each tuple in the outer table, compare it with each tuple in the inner table. This is the worst case scenario where the DBMS must do an entire scan of the inner table for each tuple in the outer table without any caching or access locality.

**Cost:**  $M + (m \times N)$

### Block Nested Loop Join

For each block in the outer table, fetch each block from the inner table and compare all the tuples in those two blocks. This algorithm performs fewer disk access because the DBMS scans the inner table for every outer table block instead of for every tuple.

**Cost:**  $M + (M \times N)$

If the DBMS has  $B$  buffers available to compute the join, then it can use  $B - 2$  buffers to scan the outer table. It will use one buffer to scan the inner table and one buffer to store the output of the join.

**Cost:**  $M + \left( \left\lceil \frac{M}{B-2} \right\rceil \times N \right)$

### Index Nested Loop Join

The previous nested loop join algorithms perform poorly because the DBMS has to do a sequential scan to check for a match in the inner table. However, if the database already has an index for one of the tables on the join key, it can use that to speed up the comparison. The DBMS can either use an existing index or build a temporary one for the join operation.

The outer table will be the one without an index. The inner table will be the one with the index.

Assume the cost of each index probe is some constant value  $C$  per tuple.

**Cost:**  $M + (m \times C)$

## 3 Sort-Merge Join

---

At a high-level, a sort-merge join sorts the two tables on their join key(s). The DBMS can use the external mergesort algorithm for this. It then steps through each of the tables with cursors and emits matches (like in mergesort).

This algorithm is useful if one or both tables are already sorted on join attribute(s) (like with a clustered index) or if the output needs to be sorted on the join key anyways.

The worst case scenario for this algorithm is if the join attribute for all the tuples in both tables contain the same value, which is very unlikely to happen in real databases. In this case, the cost of merging would be  $M \cdot N$ . Most of the time though, the keys are mostly unique so the merge cost is approximately  $M + N$ .

Assume that the DBMS has  $B$  buffers to use for the algorithm:

- Sort Cost for Table  $R$ :  $2M \times 1 + \lceil \log_{B-1} \lceil \frac{M}{B} \rceil \rceil$
- Sort Cost for Table  $S$ :  $2N \times 1 + \lceil \log_{B-1} \lceil \frac{N}{B} \rceil \rceil$
- Merge Cost:  $(M + N)$

**Total Cost: Sort + Merge**

## 4 Hash Join

The high-level idea of the hash join algorithm is to use a hash table to split up the tuples into smaller chunks based on their join attribute(s). This reduces the number of comparisons that the DBMS needs to perform per tuple to compute the join. Hash join can only be used for equi-joins on the complete join key.

If tuple  $r \in R$  and a tuple  $s \in S$  satisfy the join condition, then they have the same value for the join attributes. If that value is hashed to some value  $i$ , the  $R$  tuple has to be in bucket  $r_i$ , and the  $S$  tuple has to be in bucket  $s_i$ . Thus, the  $R$  tuples in bucket  $r_i$  need only to be compared with the  $S$  tuples in bucket  $s_i$ .

### Basic Hash Join

- **Phase #1 – Build:** First, scan the outer relation and populate a hash table using the hash function  $h_1$  on the join attributes. The key in the hash table is the join attributes. The value depends on the implementation (can be full tuple values or a tuple id).
- **Phase #2 – Probe:** Scan the inner relation and use the hash function  $h_1$  on each tuple's join attributes to jump to the corresponding location in the hash table and find a matching tuple. Since there may be collisions in the hash table, the DBMS will need to examine the original values of the join attribute(s) to determine whether tuples are truly matching.

If the DBMS knows the size of the outer table, the join can use a static hash table. If it does not know the size, then the join has to use a dynamic hash table or allow for overflow pages.

### Grace Hash Join / Hybrid Hash Join

When the tables do not fit on main memory, the DBMS has to swap tables in and out essentially at random, which leads to poor performance. The Grace Hash Join is an extension of the basic hash join that also hashes the inner table into partitions that are written out to disk.

- **Phase #1 – Build:** First, scan both the outer and inner tables and populate a hash table using the hash function  $h_1$  on the join attributes. The hash table's buckets are written out to disk as needed. If a single bucket does not fit in memory, the DBMS can use *recursive partitioning* with different hash function  $h_2$  (where  $h_1 \neq h_2$ ) to further divide the bucket. This can continue recursively until the buckets fit into memory.
- **Phase #2 – Probe:** For each bucket level, retrieve the corresponding pages for both outer and inner tables. Then, perform a nested loop join on the tuples in those two pages. The pages will fit in memory, so this join operation will be fast.

Partitioning Phase Cost:  $2 \times (M + N)$

Probe Phase Cost:  $(M + N)$

**Total Cost:**  $3 \times (M + N)$